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SEMINARIO DE GEOMETRÍA ALGEBRAICA

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Impartirá la conferencia

A sharper estimate on the Betti numbers of sets defined by quadratic inequalities

Summary: In this talk we consider the problem of bounding the Betti numbers, $b_i(S)$, of a semi-algebraic set $S \subset \mathbb{R}^k$ defined by polynomial inequalities $P_1 \geq 0, \ldots, P_s \geq 0$, where $P_i \in \mathbb{R}[X_1, \ldots, X_k]$ and $\deg(P_i) \leq 2$, for $1 \leq i \leq s$. We prove that for $0 \leq i \leq k-1$,

$$b_i(S) \leq \frac{1}{2}\left(\sum_{j=0}^{min\{s,k-i\}} \binom{s}{j}\binom{k+1}{j}2^j\right).$$

In particular, for $2 \leq s \leq \frac{k}{2}$, we have

$$b_i(S) \leq \frac{1}{2}3^s\binom{k+1}{s} \leq \frac{1}{2}\left(\frac{3e(k+1)}{s}\right)^s.$$

This improves the bound of $k^{O(s)}$ proved by Barvinok. This improvement is made possible by a new approach, whereby we first bound the Betti numbers of non-singular complete intersections of complex projective varieties defined by generic quadratic forms, and use this bound to obtain bounds in the real semi-algebraic case. We will introduce all the necessary notations and previous results.